Parked Vehicle Assisted VFC System with Smart Parking: An Auction Approach

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***Abstract*—Vehicular fog computing (VFC) is a promising approach to provide ultra-low-latency service to vehicles and end users by extending the fog computing to conventional vehicular networks. Parked vehicle assistance (PVA), as a critical technique in VFC, can be integrated with smart parking in order to exploit its full potentials. In this paper, we propose a VFC system by combining both PVA and smart parking. A single- round multi-item parking reservation auction is proposed to guide the on-the-move vehicles to the available parking places with less effort and meanwhile exploit the fog capability of parked vehicles to assist the delay-sensitive computing services. The proposed allocation rule maximizes the aggregate utility of the smart vehicles and the proposed payment rule guarantees incentive compatible, individual rational and budget balance. The simulation results confirmed the win-win performance enhancement to fog node controller (FNC), vehicles, and parking places from the proposed design.**

I. Introduction

With the rapid growth of connected devices in Internet of Things (IoT)-based network systems and new applications and services for 5G, such as virtual reality (VR), augmented reality (AR) and real-time online gaming, fog computing plays an important a role to provide low latency data service by offloading the urgent computation workload from cloud date centers. Vehicular fog computing (VFC) is one of the potential applications by extending the fog computing to conventional vehicular networks [1]. To cope with the explosive application demands, roadside units (RSUs), which are generally deployed in different areas of a city, can easily be upgraded by equipped with fog computing servers to provide both communication and computation services to those mobile terminals. However, the fog computing service is limited due to the density of RSUs. Moreover, RSUs confront heavy load with the increasing number of service requests. The idea of parked vehicle assistance (PVA) has been investigated to be useful to deliver content in vehicular ad hoc networks (VANETs) where the number of RSUs is insufficient [2–6]. Note that by sharing and exchanging contents with moving vehicles, the parked vehicles act as static infrastructures to improve connectivity. The exploit of parked vehicles as infrastructures for both communication and computation is recognized as an important component of the future VFC systems [7].

On the other hand, due to the increase population and spatial resource of a city, limited parking places cause severe parking issues. Previous study shows that a large portion of traffic intensity in major city is due to the congestion caused by the vehicles searching for a parking slot [8]. Besides, unnecessary time and energy of vehicles is wasted during their searching for parking. To guide the vehicles to the available parking slots with less effort, time and fuel consumption, smart parking system (SPS) has been widely investigated [9]. Nevertheless, most proposed designs are limited to solely satisfy the parking demands. The potential benefits of smart parking in other domains, such as VFC, is not explored yet.

We find that by integrating PVA and smart parking, we potentially provide a more robust VFC system with lower cost and higher satisfactory to all participants. Consider that smart parking with reservation (pre-booked) not only benefits to mitigate the traffic congestion and reduce unnecessary driving expenses but also offers an opportunity to lead moving vehicles to the available parking places nearby the areas where the delay-sensitive computing services are not inadequate. The vehicles with fog capability could be attracted to park at proper parking spaces and provide fog services through smart parking price and/or additional compensations from fog service provider. Moreover, the network operator can save a part of deployment expense and maintenance cost with the aid of parked vehicles.

In this paper, we propose a practical VFC system by combining both PVA and smart parking. A single-round multi-item parking reservation auction is provide to guide the on-the-move vehicles to the available parking places with less effort and meanwhile exploit the fog capability of parked vehicles to assist the delay-sensitive computing services. The proposed system motivates the parked vehicles by paying a certain amount of monetary rewards to compensate their service cost. The proposed allocation rule maximizes the aggregate utility of the smart vehicles and the proposed payment rule guarantees incentive compatible, individual rational and budget balance.

Related Work

The potential of PVA is investigated in the literature. The results in [2] show that even a small proportion of PVA vehicles can greatly promote the network connectivity. Theoretic analysis, realistic survey and simulation of PVA

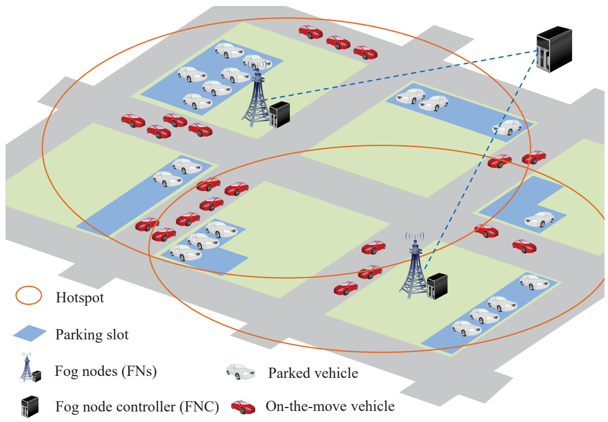


Fig. 1. Local network scenario

are investigated in [3]. The fairness in exploiting the energy resources of parked vehicles is considered in [4] to extend the RSU service coverage, which is constrained to not excessively drain parked vehicle batteries. D2D-based content delivery is proposed in [5], where parked vehicles around the street form vehicular social communities with the moving vehicles passing along the road through D2D communications. A Stackelberg game has been developed to obtain the equilibrium of the competition and cooperation among RSUs, moving vehicles and parked vehicles during the content delivery [6].

A dynamic resource allocation, reservation, and pricing smart parking system is proposed to minimize the overall system cost [10]. However, the uncoordinated selfish behavior of drivers is not addressed and will potentially degrades the system efficiency. A demand-based parking pricing mechanism is proposed by predicting the occupancy rate of individual parking areas using machine learning approach [11], where an amount of historical parking data are required.

To best of our knowledge, the idea of combining both VFC and smart parking is not explored in the literature yet.

II. Problem Formulation

The proposed parked vehicle assisted VFC system is shown in Fig. 1. In the system, we assume a set of $\mathcal{H} = \{h_1, h_2, \cdots, h_K\}$ to represent K hotspots. Each hotspot consists of multiple end-user devices carrying with delay-sensitive computation requests for external computing service. To guarantee the service requirements, sufficient fog nodes (FNs) have been deploy near each hotspot to serve those end users. A fog node controller (FNC) is considered to manage those FNs. Besides, we assume a set of $\mathcal{B} = \{b_1, b_2, \cdots, b_N\}$ to denote total N *on-the-move* vehicles (moving on the road with certain trip destinations). Some of them equipped with limited computing resources, called *fog-capable vehicles*, have potential to offload the computation workload from the FNs near the hotspots when they are also parked nearby. There are M parking places own by private parking operators, denoted by $\mathcal{S} = \{s_1, s_2, \cdots, s_M\}$, which provide parking service to those vehicles searching for parking slots. The real-time information about parking availability can be collected by parking sensors. We consider a time-slotted system and formulate as follows:

1) Parking places: We introduce $x_{i,j}(t)$ to indicate the arrive event for parking:

$$x_{i,j}(t) = \begin{cases} 1 & \text{if } b_i \text{ park at } s_j \text{ at period } t, \\ 0 & \text{otherwise.} \end{cases} \quad (1)$$

The departure event is denoted by $y_{i,j}(t)$ as

$$y_{i,j}(t) = \begin{cases} 1 & \text{if } b_i \text{ parked at } s_j \text{ leave at period } t, \\ 0 & \text{otherwise.} \end{cases} \quad (2)$$

For parking place $s_j \in \mathcal{S}$, the geo-location is known as $\mathcal{X}_j^{\mathrm{s}} \in \mathbb{R}^2$ (GPS coordinates). We denote C_j as the overall parking capacity of s_j. Then, the parking slot inventory at period t can be calculated by

$$C_j(t) = C_j - \sum_{b_i \in \mathcal{B}} \sum_{t'=1}^{t-1} x_{i,j}(t') + \sum_{b_i \in \mathcal{B}} \sum_{t'=1}^{t} y_{i,j}(t'). \quad (3)$$

2) Vehicles: We introduce a binary indicator $w_{i,j}$: If vehicle b_i stay at place s_j at period t then $w_{i,j} = 1$; Otherwise, $w_{i,j} = 0$. Therefore, those vehicles ($b_i \in \mathcal{B}$) with the condition $\sum_{s_j \in \mathcal{S}} w_{i,j} = 0$ are on-the-move vehicle.

For on-the-move vehicle $b_i \in \mathcal{B}$ searching for a parking slot, the current car position and the traveling destination are known as $\mathcal{X}_i^{\mathrm{cur}}$ and $\mathcal{X}_i^{\mathrm{dest}}$, respectively. Vehicle b_i can measure its average driving speed r_i^{d} through the historical information. In this paper, we assume that the average walking speed of human is r^{w} in general. Besides, we define $\Phi^{\mathrm{r}}(\cdot)$ as the distance function by city roads (similar to Manhattan distance). If vehicle b_i determines to park at place s_j, the remaining driving time and the walking time will be

$$\tau_{i,j}^{\mathrm{d}} = \Phi^{\mathrm{r}}(\mathcal{X}_i^{\mathrm{cur}}, \mathcal{X}_j^{\mathrm{s}})/r_i^{\mathrm{d}} \text{ and } \tau_{i,j}^{\mathrm{w}} = \Phi^{\mathrm{r}}(\mathcal{X}_j^{\mathrm{s}}, \mathcal{X}_i^{\mathrm{dest}})/r^{\mathrm{w}}. \quad (4)$$

Therefore, the total traveling time will be $\tau_{i,j} = \tau_{i,j}^{\mathrm{d}} + \tau_{i,j}^{\mathrm{w}}$. We define the driving energy cost as $c_{i,j}^{\mathrm{d}} = \theta\tau_{i,j}^{\mathrm{d}}$, where θ is the per unit driving energy cost. The total cost of both driving and walking can be estimated by

$$c_{i,j} = c_{i,j}^{\mathrm{d}} + \delta\tau_{i,j}, \quad (5)$$

where δ is a positive constant translates traveling time to cost.

Furthermore, we define CPU as the unit amount of computing resource, which has the service rate μ (in CPU cycle per bit). A vehicle b_i can provide computing service if $m_i \neq 0$, where m_i is the number of CPUs equipped at b_i and reflects its fog capability. We assume that a parked vehicle will only serve at most one hotspot during its parking duration Δ_i.

3) Hotspots: For hotspot $h_k \in \mathcal{H}$, the central geo-location is known as $\mathcal{X}_k^{\mathrm{h}}$. We assume that the FNC is capable to predict the amount of computing service requests from end users at h_k for a short period in the future, which is represented by mean workload arrival rate λ_k. We allocate the workload to each CPU evenly. That is, we have $\hat{\lambda}_k = \lambda_k/m_k$, where $\hat{\lambda}_k$ is the offloaded arrival rate per CPU and m_k is the given total number of CPUs required by hotspot h_k by considering its delay requirements.

We introduce a binary indicator as follows:

$$z_{i,k} = \begin{cases} 1 & \text{if } b_i \text{ provide service to } h_k, \\ 0 & \text{otherwise.} \end{cases} \tag{6}$$

If vehicle b_i provides computing service to h_k ($z_{i,k} = 1$) at s_j ($w_{i,j} = 1$), the quality-of-service (QoS) can be measured in terms of service delay [12]:

$$d_{i,j}^k = q_{i,j}^k + h_{i,j}^k, \tag{7}$$

which consists of the queuing delay $q_{i,j}^k$ (CPU load) and the network delay $h_{i,j}^k$ (data deliver). Consider parallel M/G/1 processor sharing queues [13, 14], the queuing delay is

$$q_{i,j}^k = \frac{\hat{\lambda}_k m_{i,j}}{\mu - \frac{\hat{\lambda}_k m_{i,j}}{m_{i,j}}} = \frac{\hat{\lambda}_k m_{i,j}}{\mu - \hat{\lambda}_k}, \tag{8}$$

where $m_{i,j}$ is the number of CPUs provided by b_i at s_j. According to [12], the network delay is defined as

$$h_{i,j}^k = h_{j,k} = \xi \Phi^{\mathrm{l}}(\mathcal{X}_j^{\mathrm{s}}, \mathcal{X}_k^{\mathrm{h}}), \tag{9}$$

where $h_{j,k}$ is the network delay between the parking place s_j and the hotspot h_k, ξ is a scalar and $\Phi^{\mathrm{l}}(\cdot)$ is the air line distance function. Furthermore, we define the energy cost of computing service [15] as

$$c_{i,j}^k = \alpha \frac{\hat{\lambda}_k m_{i,j}^2}{\mu} + \beta \Phi^{\mathrm{l}}(\mathcal{X}_k^{\mathrm{h}}, \mathcal{X}_j^{\mathrm{s}}). \tag{10}$$

Furthermore, we define D_k as the maximum delay toleration of h_k. That is to say, the parked vehicle b_i can serve hotspot h_k at s_j only when $d_{i,j}^k \le D_k$.

We assume that the FNC manages a sufficient number of nearby FNs to meet the service requirements. An energy cost function $C(m_k^f)$ is defined, where m_k^f is the number of CPUs required to be turned on at nearby FNs of hotspot h_k:

$$m_k^f = m_k - \sum_{b_i \in \mathcal{B}} \sum_{s_j \in \mathcal{S}} w_{i,j} z_{i,k} m_{i,j}. \tag{11}$$

Note that $C(m_k^f)$ is a strictly convex and increasing function subject to m_k^f. Without loss of generality, we define $C(m_k^f)$ as

$$C(m_k^f) = \alpha^f \frac{\hat{\lambda}_k (m_k^f)^2}{\mu}, \tag{12}$$

which is similar to equation (10) and the network cost is ignored.

So far, we have described all the mathematical formulations. Consider that the FNC can achieve energy saving by turning off redundant FNs when the computation workload from the FNs near the hotspots is offloaded by those vehicles pared at the parking places. To attract the fog-capable vehicles carried with CPU resources to the expected parking places, the FNC needs to pay a certain amount of monetary rewards for service offloading. The goal of the FNC is to minimize its cost of satisfying the demands of all end-users in all hotspots under the constraint that the profit of each hotspot should be non-negative, which is represented by its cost saving by offloading workloads to parked vehicles minus the total offload payments. Each on-the-move vehicle, on the other hand, aims to maximize its own utility when requesting for parking reservation service. They may compete with each other for preferred parking slots. It motivates us to employ a parking reservation auction to regulate the proposed VFC system.

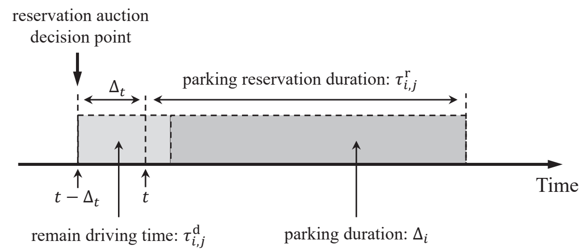


Fig. 2. Time sequence of parking reservation

III. Auction Design

In this section, a single-round multi-item parking reservation auction is presented to guide the on-the-move vehicles to the available parking places and meanwhile regulate the parked vehicles to assist the delay-sensitive computing services. We provide the strategies of the smart vehicles. We also devise an allocation rule and a payment rule to guarantee the desired economic properties, such as incentive compatible and individual rationality.

A. Auction Model

The FNC acts as a *auctioneer*, who periodically holds the parking reservation auction. An offload price $a_{j,k}$ is announced for per CPU resource provided by parked vehicles at s_j to hotspot h_k. The parking place operators are *sellers* and each of them provides homogeneous goods, saying the unoccupied parking slots. A reserve price p_j^s (minimum acceptable) is charged per unit time if the vehicle choose to park at s_j. The on-the-move vehicles, which act as *buyers / bidders*, request for parking reservation service. They aim to maximize the own utility by considering cost, price, proximity, service income, etc.

We define a reservation deadline Δ_t ahead of parking reservation point, as shown in Fig. 2. The reservation auction is formulated on a rolling horizon of Δ_t intervals an starts at the decision point $(t - \Delta_t)$ periodically. If vehicle b_i chooses to park at s_j, the parking reservation duration will be

$$\tau_{i,j}^{\mathrm{r}} = \tau_{i,j}^{\mathrm{d}} + \Delta_i - \Delta_t. \tag{13}$$

Then, the parking duration between arrives and departs of b_i is known as $[T_{i,j}^{\mathrm{in}}, T_{i,j}^{\mathrm{out}}]$, where $T_{i,j}^{\mathrm{in}} = t - \Delta_t + \tau_{i,j}^{\mathrm{d}}$ and $T_{i,j}^{\mathrm{out}} = T_{i,j}^{\mathrm{in}} + \Delta_i$.

The proposed reservation auction is presented as follows:

- *Step 1*: On-the-move vehicles request for parking reservation service to the FNC.
- *Step 2*: The FNC collects the occupancy information $C_j(t)$ and reserve prices p_j^{s} from parking places. And then, the FNC announces the offload price $a_{j,k}$, reserve price p_j^s to the smart vehicles.
- *Step 3*: The vehicles submit the bidding vectors to the FNC.
- *Step 4*: The FNC applies predefined *allocation rule* and *payment rule* to determine the parking allocation and the corresponding parking payment.

B. Strategies

1) Vehicles: Let v_i denote the value of successful parking to b_i. The utility of b_i parking at s_j is

$$u_{i,j} = v_{i,j} - \tau_{i,j}^{\mathrm{r}} p_j^s, \qquad \text{where} \tag{14}$$

$$v_{i,j} = \begin{cases} v_i - c_{i,j} + \sum\limits_{h_k \in \mathcal{H}} z_{i,k} \Delta_i (a_{j,k} m_{i,j} - c_{i,j}^k) \\ \qquad \text{if } \tau_{i,j}^{\mathrm{d}} \geq \Delta_t,\ \tau_{i,j}^{\mathrm{w}} \leq T_i^{\mathrm{w}} \text{ and } \tau_{i,j} \leq T_i, \\ 0 \qquad \text{otherwise,} \end{cases} \tag{15}$$

is the true valuation. T_i^{w} and T_i are the maximum tolerant walking time (maximum distance b_i would like to walk from the reserved parking place to the destination) and the maximum tolerant traveling time of b_i, respectively. $v_{i,j}$ is regarded as the bidding of b_i to s_j. Therefore, the set of smart vehicles is defined as $\mathcal{B}_{\mathrm{b}} = \{b_i \mid v_{i,j} > 0, \exists s_j\}$. Besides, we denote $\mathcal{H}_j$ as the candidate hotspots served by the parked vehicles at s_j, where $h_{j,k} < D_k$ for $\forall h_k \in \mathcal{H}_j$. Accordingly, the candidate parking places of hotspot h_k is denoted by $\mathcal{S}_k$, where $h_k \in \mathcal{H}_j$ for $\forall s_j \in \mathcal{S}_k$.

The amount of CPU resource vehicle b_i would contribute to the FNC will depend on the benefit he/she may gain minus the cost. A hard constraint is that the service delay cannot surpass the maximum delay toleration D_k, that is,

$$\begin{aligned} d_{i,j}^k \leq D_k \Rightarrow \quad & \frac{\hat{\lambda}_k m_{i,j}}{\mu - \hat{\lambda}_k} + \xi \Phi^{\mathrm{l}}(\mathcal{X}_j^{\mathrm{s}}, \mathcal{X}_k^{\mathrm{h}}) \leq D_k \\ \Rightarrow \quad & m_{i,j} \leq \frac{(\mu - \hat{\lambda}_k)(D_k - \xi \Phi^{\mathrm{l}}(\mathcal{X}_j^{\mathrm{s}}, \mathcal{X}_k^{\mathrm{h}}))}{\hat{\lambda}_k}. \end{aligned} \tag{16}$$

To achieve the maximum utility when parking at s_j, b_i needs to select its serving hotspot and the corresponding optimal number of offered CPUs:

$$\begin{aligned} & \{k^*, m_{i,j}^*\} = \arg\max_{m_{i,j}} (a_{j,k} m_{i,j} - c_{i,j}^k), k \in \mathcal{H}_j \\ \text{s.t.} \quad & m_{i,j} \in \{1, \cdots, m_i\}, \\ & m_{i,j} \leq \frac{(\mu - \hat{\lambda}_k)(D_k - \xi \Phi^{\mathrm{l}}(\mathcal{X}_j^{\mathrm{s}}, \mathcal{X}_k^{\mathrm{h}}))}{\hat{\lambda}_k}, \forall h_k \in \mathcal{H}_j \end{aligned} \tag{17}$$

Note that it is possible for b_i that any offer will give it negative utility due to insufficient compensation from the FNC. In such a case, b_i chooses to park at s_j without providing offload service. Therefore, the bidding vector of b_i to s_j is represented as $\{v_{i,j}, k^*, m_{i,j}^*\}$.

2) FNC: With the service aid of parked vehicles, the cost saving at h_k will be

$$\Delta_k^{\mathrm{cost}} = C(m_k) - C(m_k^f). \tag{18}$$

Therefore, the FNCs instantaneous profit is represented as

$$\sum_{h_k \in \mathcal{H}} \Delta_k^{\mathrm{cost}} - \sum_{b_i \in \mathcal{B}} \sum_{s_j \in \mathcal{S}_i} \sum_{h_k \in \mathcal{H}} w_{i,j} z_{i,k} m_{i,j} a_{j,k}. \tag{19}$$

We consider non-negative profit for each hotspot, that is,

$$\begin{aligned} & \Delta_k^{\mathrm{cost}} - \sum_{b_i \in \mathcal{B}} \sum_{s_j \in \mathcal{S}_i} w_{i,j} z_{i,k} m_{i,j} a_{j,k} \\ =& 2\alpha \frac{\hat{\lambda}_k m_k}{\mu} \sum \sum w_{i,j} z_{i,k} m_{i,j} \\ & - \alpha \frac{\hat{\lambda}_k}{\mu} (\sum \sum w_{i,j} z_{i,k} m_{i,j})^2 - \sum \sum w_{i,j} z_{i,k} m_{i,j} a_{j,k} \\ \geq & 2\alpha \frac{\hat{\lambda}_k m_k}{\mu} \sum \sum w_{i,j} z_{i,k} m_{i,j} \\ & - \alpha \frac{\hat{\lambda}_k m_k}{\mu} \sum \sum w_{i,j} z_{i,k} m_{i,j} - \sum \sum w_{i,j} z_{i,k} m_{i,j} a_{j,k} \\ =& \sum \sum w_{i,j} z_{i,k} m_{i,j} (\alpha \frac{\hat{\lambda}_k m_k}{\mu} - a_{j,k}) \geq 0. \end{aligned} \tag{20}$$

Therefore, we have

$$a_{j,k} \leq \alpha \frac{\hat{\lambda}_k m_k}{\mu}. \tag{21}$$

On the other hand, to attract those fog-capable vehicles that park at s_j and meanwhile provide computing services to hotspot h_k, the FNC needs to guarantee the constraint $a_{j,k} m_{i,j} - c_{i,j}^k > 0$, that is,

$$a_{j,k} m_{i,j} - \alpha \frac{\hat{\lambda}_k m_{i,j}^2}{\mu} - \beta \Phi^{\mathrm{l}}(\mathcal{X}_k^{\mathrm{h}}, \mathcal{X}_j^{\mathrm{s}}) > 0. \tag{22}$$

Note that the quadratic function in the above constraint is an upside-down parabola with respect to $m_{i,j}$. The FNC needs to guarantee its discriminant as follows:

$$\Delta = a_{j,k}^2 - \frac{4\alpha\beta \hat{\lambda}_k \Phi^{\mathrm{l}}(\mathcal{X}_k^{\mathrm{h}}, \mathcal{X}_j^{\mathrm{s}})}{\mu} > 0. \tag{23}$$

That is to say, the offload price should satisfy

$$a_{j,k} > \sqrt{\frac{4\alpha\beta \hat{\lambda}_k \Phi^{\mathrm{l}}(\mathcal{X}_k^{\mathrm{h}}, \mathcal{X}_j^{\mathrm{s}})}{\mu}}. \tag{24}$$

So far, the FNC can announce fixed offload price $a_{j,k}$ subject to the constraints (21)(24).

C. Allocation Rule

The objective of the allocation rule is to maximize the aggregate utility of the smart vehicles in $\mathcal{B}_{\mathrm{b}}$:

$$
\begin{aligned}
w_{i,j}^{*} = \arg\max_{w_{i,j}} & \sum_{b_i\in\mathcal{B}_{\mathrm{b}}}\sum_{s_j\in\mathcal{S}} w_{i,j}(v_{i,j}-\tau_{i,j}^{\mathrm{r}}p_j^s),\\
\text{s.t.}\quad & w_{i,j}\in\{0,1\},\forall b_i\in\mathcal{B}_{\mathrm{b}},\forall s_j\in\mathcal{S}\\
& \sum_{s_j\in\mathcal{S}} w_{i,j}\le 1,\forall b_i\in\mathcal{B}_{\mathrm{b}}\\
& \sum_{b_i\in\mathcal{B}_{\mathrm{b}}} w_{i,j}\le C_j(t),\forall s_j\in\mathcal{S}\\
& w_{i,j}(v_{i,j}-\tau_{i,j}^{\mathrm{r}}p_j^s)\ge 0
\end{aligned}
\tag{25}
$$

Lemma 1. *The allocation problem* (25) *can be modified as a maximum weight b-matching problem in a weighted bipartite graph (WBG).*

Proof. In a *weighted bipartite graph*, the vertices can be decomposed into two disjoint sets U and V such that every edge with an associated weight connects a vertex in U to the other in V. A maximum weighted b-matching problem in a WBG is defined as a matching where the sum of the weights of all edges in the matching has a maximal value and each vertex in U matches at least 1 and at most $b(v)$ vertices in V. In the allocation problem (25), we construct a WBG $G=(U,V,E)$ with two disjoint sets, the set of parking places $U=\mathcal{S}$ and the set of smart vehicles $V=\mathcal{B}_{\mathrm{b}}$. The weight $A(b_i,s_j)$ of the edge connecting s_j and b_i represents the utility of b_i if parks at s_j. Parking place s_j matches at most $C_j(t)$ smart vehicles since its parking slot inventory at period t is $C_j(t)$. And a smart vehicle can be assigned to one parking slot. In this way, the problem (25) is modified as a maximum weight b-matching problem in WBG. □

Lemma 2. *The allocation problem* (25) *can be transformed into a maximum weight perfect bipartite matching (MWPBM) problem.*

Proof. To fits classic 1-matching problem, we duplicate the original WBG $G=(U,V,E)$ to a new WBG $G'=(U',V,E')$, where $U'=\{U_1,\cdots,U_M\}$, $U_j=\{s_{j,1},\cdots,s_{j,C_j(t)}\}$ and $s_{j,n}$ is the n-th parking slot of parking place s_j. An identical weight is assigned between the parking slots in the same place and a vertex in V, that is, $A(b_i,s_{j,n})=A(b_i,s_j)$. Furthermore, we transform G' to a complete WBG $G''=(U'',V',E'')$ by adding some virtual vertexes and virtual edges so that $|U''|=|V'|=\max\{\sum_{s_j\in\mathcal{S}}C_j(t),|\mathcal{B}_{\mathrm{b}}|\}$ and every vertex in U'' is connected to every vertex in V'. For those virtual vertexes, the weighted of the link is changed to be zero. In this way, we aims to find a matching of G'' where every vertex in $(U''\cup V')$ is incident to exactly one edge. Therefore, the allocation problem (25) become a MWPBM problem. □

We know that classic Kuhn-Munkres (KM) algorithm (also known as Hungarian algorithm) [16, 17] can be exploited to solve MWPBM problem, which has the complexity of $O(N^3)$. Therefore, the allocation problem (25) can be solved with the complexity of $O(N^3)$.

D. Payment Rule

The goal is to find a payment rule that satisfies the desired economic properties in a socially optimal manner subject to the reserve price constraint. Vickrey Clarke Groves (VCG) mechanism is widely used in auction design that gives bidders an incentive to bid their true valuations. We modify the VCG mechanism with Clarke pivot payments [18]. Let $\{w_{i,j}\}$ and $\{\hat{w}_{i,j}\}$ be the allocation results of problem (25) with and without b_i's participation, respectively. Then, the payment for b_i can formally be written as

$$
p_i^b=\begin{cases}
\tau_{i,j}^{\mathrm{r}}p_j^s+\sum\limits_{b_l\neq b_i}\sum\limits_{s_o\in\mathcal{S}}\hat{w}_{l,o}(v_{l,o}-\tau_{l,o}^{\mathrm{r}}p_o^s)\\
\qquad -\sum\limits_{b_l\neq b_i}\sum\limits_{s_o\in\mathcal{S}}w_{l,o}(v_{l,o}-\tau_{l,o}^{\mathrm{r}}p_o^s)\\
\qquad\qquad \text{if } b_i \text{ wins a parking reservation at } s_j,\\
0, \qquad \text{if } b_i \text{ loses the auction.}
\end{cases}
\tag{26}
$$

In this way, the reserve price required by the parking places can be guaranteed.

E. Economic Analysis

Theorem 1. *The proposed auction is incentive compatible (truthful for bidders).*

Proof. In a truthful auction, all bidders are incited to voluntarily reveal their true valuation for the items they are bidding. Let V_i be the true valuation vector of b_i and V_{-i} be the valuation vectors of other smart vehicles $\mathcal{B}_{\mathrm{b}}\backslash\{b_i\}$. When V_i and V_{-i} are submitted, the utility of b_i can be calculated using the payment rule (26):

$$
\begin{aligned}
u_i &= \sum_{s_j\in\mathcal{S}} w_{i,j}v_{i,j}-p_i^b\\
&= \sum_{s_j\in\mathcal{S}} w_{i,j}(v_{i,j}-\tau_{l,j}^{\mathrm{r}}p_j^s)+\sum_{b_l\neq b_i}\sum_{s_o\in\mathcal{S}} w_{l,o}(v_{l,o}-\tau_{l,o}^{\mathrm{r}}p_o^s)\\
&\qquad -\sum_{b_l\neq b_i}\sum_{s_o\in\mathcal{S}}\hat{w}_{l,o}(v_{l,o}-\tau_{l,o}^{\mathrm{r}}p_o^s).
\end{aligned}
\tag{27}
$$

When V_i' and V_{-i} are submitted, the utility of b_i is

$$
\begin{aligned}
u_i' &= \sum_{s_j\in\mathcal{S}} w_{i,j}'v_{i,j}-p_i^b\\
&= \sum_{s_j\in\mathcal{S}} w_{i,j}'(v_{i,j}-\tau_{l,j}^{\mathrm{r}}p_j^s)+\sum_{b_l\neq b_i}\sum_{s_o\in\mathcal{S}} w_{l,o}'(v_{l,o}-\tau_{l,o}^{\mathrm{r}}p_o^s)\\
&\qquad -\sum_{b_l\neq b_i}\sum_{s_o\in\mathcal{S}}\hat{w}_{l,o}(v_{l,o}-\tau_{l,o}^{\mathrm{r}}p_o^s)
\end{aligned}
\tag{28}
$$

Since $\{w_{i,j}\}$ maximizes the total profit defined by equation (25), we have

$$
\begin{aligned}
&\sum_{s_j\in\mathcal{S}} w_{i,j}(v_{i,j}-\tau_{l,j}^{\mathrm{r}}p_j^s)+\sum_{b_l\neq b_i}\sum_{s_o\in\mathcal{S}} w_{l,o}(v_{l,o}-\tau_{l,o}^{\mathrm{r}}p_o^s)\\
&\ge \sum_{s_j\in\mathcal{S}} w_{i,j}'(v_{i,j}-\tau_{l,j}^{\mathrm{r}}p_j^s)+\sum_{b_l\neq b_i}\sum_{s_o\in\mathcal{S}} w_{l,o}'(v_{l,o}-\tau_{l,o}^{\mathrm{r}}p_o^s).
\end{aligned}
\tag{29}
$$

TABLE I
LIST OF KEY SIMULATION PARAMETERS

α^f, α, β, ξ	0.8, 0.6, 0.4, 0.001
δ, θ	10, \$0.5/km
C_j, p_j^s	unif(5, 20), unif(\$1, \$5)
μ, λ_k	1MB/s, unif(5MB/s, 20MB/s)
m_i, m_k	unif(0, 5), 200
D_k	unif(10ms, 50ms)
r^{w}, r_i^{d}	3.5km/h, unif(10km/h, 30km/h)
Δ_t, Δ_i	0.01h, unif(0.5h, 5h)
T_i^{w}, T_i	0.3h = 18min, 0.5h = 30min

By subtracting the term $\sum_{b_l \neq b_i} \sum_{s_o \in \mathcal{S}} \hat{w}_{l,o}(v_{l,o} - \tau_{l,o}^{\mathrm{r}} p_o^s)$ from both sides of the inequality, we get $u_i \geq u_i'$, which means that the incentive compatible property is hold. □

Theorem 2. *The proposed auction is individually rational.*

Proof. The utility of the sellers and buyers should be no less than zero. Each agent participating in the auction can expect a non-negative utility. According to the payment rule (26), we observe that the per unit payment from each parked vehicle is greater than the reserved price announced by sellers since $p_i^b/\tau_{i,j}^{\mathrm{r}} \geq p_j^s$ if b_i win a parking reservation at s_j. That is, the sellers can achieve non-zero utility. Besides, we consider the utility of b_i in equation (27) and have

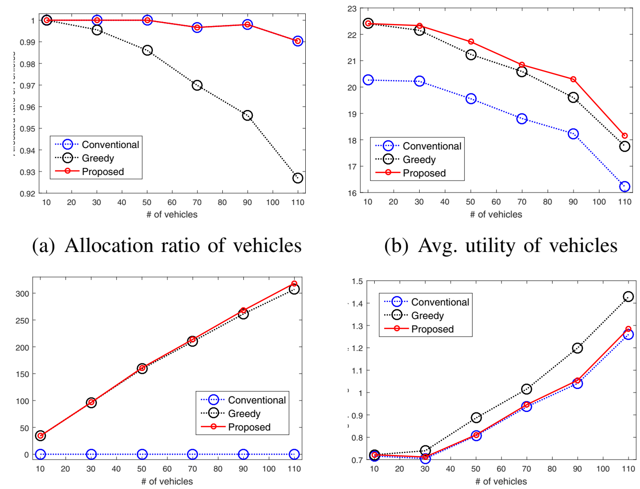
$$\begin{aligned} u_i = & \sum_{s_j \in \mathcal{S}} w_{i,j}(v_{i,j} - \tau_{l,j}^{\mathrm{r}} p_j^s) + \sum_{b_l \neq b_i} \sum_{s_o \in \mathcal{S}} w_{l,o}(v_{l,o} - \tau_{l,o}^{\mathrm{r}} p_o^s) \\ & - \sum_{b_l \neq b_i} \sum_{s_o \in \mathcal{S}} \hat{w}_{l,o}(v_{l,o} - \tau_{l,o}^{\mathrm{r}} p_o^s) \\ \geq & \sum_{b_l \in \mathcal{B}_{\mathrm{b}}} \sum_{s_o \in \mathcal{S}} w_{l,o}(v_{l,o} - \tau_{l,o}^{\mathrm{r}} p_o^s) \\ & - \sum_{b_l \in \mathcal{B}_{\mathrm{b}}} \sum_{s_o \in \mathcal{S}} \hat{w}_{l,o}(v_{l,o} - \tau_{l,o}^{\mathrm{r}} p_o^s) \geq 0, \end{aligned} \tag{30}$$

which shows individual rationality of buyers. □

Moreover, we know that the FNC can achieve non-negative profit of each hotspot when announces the offload price subject to the constraint (21). Therefore, *budget balance* is hold in the proposed auction.

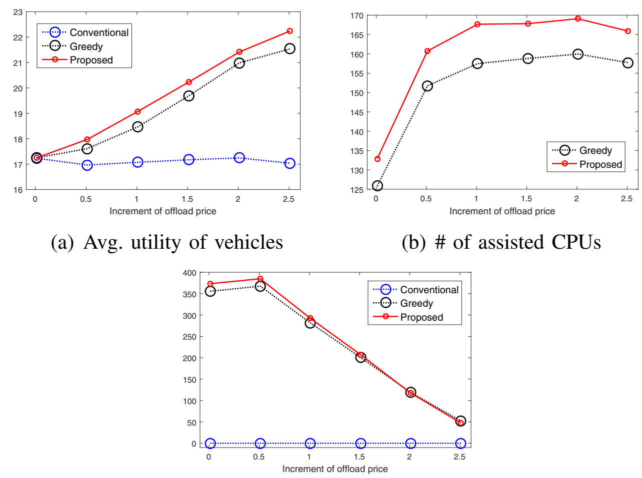
IV. SIMULATION RESULTS

In this section, simulation results are provided to verify the performance of the proposed VFC. We consider a urban map with 1km range, where parking places and hotspots are uniformly scattered. The current car position and the traveling destination are also initialized by following uniform distribution. The key simulation parameters are listed in Table I. Specifically, we assume that the parking valuation v_i is proportional to the parking duration Δ_i, saying $v_i = 10\Delta_i$ in our simulations. We compare three approaches: 1) *Conventional*, without the aid of parked vehicles; 2) *Greedy*, in which the vehicles are allocated according to greedy allocation; 3) and *Proposed*, the proposed approach. The greedy allocation rule is defined that the available parking slots is priority allocated to the smart vehicle with maximum bidding value $v_{i,j}$.



(a) Allocation ratio of vehicles
(b) Avg. utility of vehicles
(c) Profit of FNC
(d) Avg. parking income (per vehicle)

Fig. 3. Performance versus the number of vehicles: $K = 16$; $M = 8$.



(a) Avg. utility of vehicles
(b) # of assisted CPUs
(c) Profit of FNC

Fig. 4. Performance versus the increment of offload price: $K = 16$; $M = 8$; $N = 100$.

In Fig. 3, we show the performance versus the number of vehicles. Allocation ratio of vehicle in Fig. 3(a) is calculated by the number of those vehicles, which have been successfully allocated parking slots, over the number of smart vehicles. We observe that the allocation ratio decreases against the number of vehicles due to the limited parking slots. *Greedy* causes a great loss of successful parking reservation when the number of vehicles is more than 100. However, *Proposed* can satisfy almost all of smart vehicles. The decline in Fig. 3(b) indicates the intensified competition of smart vehicles for preferred parking slots. Nevertheless, *Proposed* achieves best average utility of vehicles comparing with *Conventional* and

Greedy. In Fig. 3(c), both *Proposed* and *Greedy* outperform *Conventional* in terms of FNC's profit since *Conventional* doesn't deliver any cost saving for the FNC. The maximum average parking income per vehicle is achieved by *Greedy*, as shown in Fig. 3(d), which means that the utility of the parked vehicles will be degraded accordingly as we observed in Fig. 3(b). On the other hand, *Proposed* guarantees the benefits of vehicles subject to the reserve price constraint and meanwhile provide additional income to the parking places.

We then present the performance versus the offload price in Fig. 4. The announced offload price is obtained by the constraint in equation (21) plus a increment. We observe that the average utility of vehicles increases against the offload price because the compensation from the FNC becomes larger and larger than the service cost. We also find out that higher compensation, saying the offload price, can attracts more parked vehicles to provide their CPUs in Fig. 4(b). *Proposed* outperforms *Greedy* by attracting more CPUs for service offloading and cost saving. Nevertheless, the profit of the FNC is not always increasing due to the unnecessary offload payments, as shown in Fig. 4(c). The overpriced offload payment makes no increment to the number of assisted CPUs and decreases the profit of the FNC. This suggests an optimal choice of offload price exists and deserves further study in the future.

V. Conclusion

In this paper, a single-round multi-item parking reservation auction is proposed to guide the on-the-move vehicles to the available parking places to provide PVA service while satisfying their parking demands. We theoretically prove that the fog-aware smart parking problem can be transformed to maximum weight b-matching problem and the optimal allocation can be derived in polynomial time. Given this result, we theoretically prove that the proposed auction design guarantees incentive compatible, individual rational and budget balance. The simulation results confirm the performance improvement from the proposed design comparing with conventional and greedy approaches, especially when the parking demand is huge.

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